Compound Interest

Student Worksheet

Simple vs. Compound

Case 1:
1. You invest $1,1000 in savings account that earns 3% interest for 3 years.
   a. Find the amount of simple interest that you would earn at the end of a 3-year period. [use \( P = Irt \)]
   
   b. What is the total amount of money that you will have after this 3-year period?

Case 2:
2. You invest $1,1000 in savings account that earns 3% interest for 1 year.
   a. Find the amount of simple interest that you would earn at the end of a 1-year period. [use \( P = Irt \)]
   
   b. What is the total amount of money that you will have after this 1-year period?

3. Take the total amount of money that you have from #2 part b and put it back into the same account for another year.
   a. Find the amount of simple interest that you would earn at the end of a 1-year period. [use \( P = Irt \)]
   
   b. What is the total amount of money that you will have after this 1-year period?

4. Take the total amount of money that you have from #3 part b and put it back into the same account for another year.
   a. Find the amount of simple interest that you would earn at the end of a 1-year period. [use \( P = Irt \)]
   
   b. What is the total amount of money that you will have after this 1-year period?
5. How does the total amount of money that you have after 3 years from Case 1 compare to the total amount of money you have from Case 2?

In Case 2, the interest was **compounded**, meaning that the interest is earned not only on the principal but on the interest that was previously earned as well.

6. Complete the following table showing the balance of your account using compound interest. (The 3rd and 4th columns will be blank for right now.)

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>Balance (dollars)</th>
<th>Balance in terms of previous balance</th>
<th>Balance in terms of original balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. For successive years, what percent of the previous balance is the new balance?
   
   a. Years 0 to 1: __________
   
   b. Years 1 to 2: __________
   
   c. Years 2 to 3: __________
   
   d. What do you notice about the percentages?
   
   e. Write each balance in terms of the previous balance (record in column 3).

8. Write each balance in terms of the original balance (record in column 4).

The general form for **compound interest** (an exponential growth model) is the equation:

\[
A = P\left(1 + \frac{r}{n}\right)^{nt}
\]

where, \(P\) is the principal amount, or the original amount of money before any growth occurs, \(r\) is the annual nominal interest rate or the growth rate in decimal form, \(n\) is the number of times the interest is compounded per year, \(t\) is the number of years, and \(A\) is the new amount.

Formula for Interest Compounded Annually: \(A = P(1 + r)^t\)
Formula for Interest Compounded Half Yearly: $A = P(1 + \frac{r}{2})^{2t}$

Formula for Interest Compounded Quarterly: $A = P(1 + \frac{r}{4})^{4t}$

9. What is the growth rate from Case 2?

10. You invest $650 into a savings account that earns 2.5% interest, compounded yearly. Write a model for the account balance $y$ after $t$ years.

11. Complete the following table of values showing the account balance as a function of time (years the money is in the account).

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>Account Balance (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>

12. In the space below, create a graph that shows the account balance as a function of the time.

[Before you graph, make sure to identify:
Variable quantity:
Lower Bound:
Upper Bound:
Interval: ]
13. Use the graph to determine how long you have to keep the account open in order to have
   a. a balance of $700.

   b. a balance of $800.

14. You have $1000 that you want to invest for 5 years before you use the money towards a
    large purchase. Your bank offers a simple interest rate of 3% or an annual compound
    interest rate of 2.7%. Assuming that you will leave the money in 5 years, what is the best
    way to invest with your bank?

15. Based on the previous questions, do you think compounding interest annually, half
    yearly, or quarterly will yield more or less money?

16. Calculate the annual, half yearly, and quarterly compounded interest associated with
    investing $5,000 for 4 years at an interest rate of 2.5%
    a. Annually

    b. Half Yearly

    c. Quarterly